Spatial Requirements for Linear Transducer Measurements and Excitation Point Mapping in Six-Degree-of-Freedom Vibration Testing

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Performing quality Six Degree-of Freedom (6-DOF) motion replication testing in the laboratory begins with carefully planned selection of measurement locations and orientation. This paper considers the typical scenario involving linear accelerometer reference measurements. Methods are developed to aid the test engineer in proper transducer placement and subsequent mapping of the field measurements to exciter locations. Analytical and laboratory based examples are provided.

INTRODUCTION

The integration of 6-DOF vibration testing into dynamic test laboratories is entwined with many obstacles requiring an increased level of technical skill from the test engineers planning such tests and the operators that will ultimately perform the tests. The first step in performing a 6-DOF vibration test in the laboratory certainly begins with acquiring sufficient reference data. In addition to the standard concerns related to the dynamic range and frequency response characteristics of the transducers and recording equipment used in field data acquisition phase, the quantity and spatial locations of the transducers become critical test parameters. Understanding the underlying dynamics of 6-DOF systems and the physical constraints such systems place of the spatial locations of reference transducer in order to perform true 6-DOF laboratory motion replication is essential. Similarly, it is essential that the test operator is able to understand the dynamics of an arbitrary data set that may be provided by an outside source for use as reference acceleration in a laboratory test.

The basic mathematics describing linear transducer placement requirements for 6-DOF laboratory motion replication testing is developed in the following sections. The discussion is then expanded to the mapping of the measured field data to exciter locations. Note that the following analysis is performed entirely in the time domain. This approach allows mapping of excitation requirements at the individual exciters which yield exact replication at the test article as opposed to estimating excitation levels via traditional transfer function based techniques. Detailed knowledge of the excitation requirements in the time domain eliminate any averaging of the signal thereby providing the test operator with instantaneous excitation requirements to aid in feasibility studies that are independent of the statistical properties of the reference acceleration signals.

KINEMATIC CONSIDERATIONS FOR TRANSDUCER PLACEMENT

Consider a rigid body equipped with m tri-axial transducers located as shown in Figure 1. The acceleration measured by the i^{th} transducer is given kinematically by

$$\underline{a}_i = \underline{a}_c + \underline{\dot{\omega}} \times \underline{r}_i + \underline{\omega} \times \left(\underline{\omega} \times \underline{r}_i \right), \tag{1}$$

where $\underline{\omega}$ and $\underline{\dot{\omega}}$ represent the angular velocity and angular acceleration of the rigid body, $\underline{a}_c$ represents the acceleration of some reference point in the system. In general, $\underline{a}_c$ is unknown (unless a transducer was selected *a prioi* for that location). Thus, Equation (1) represents three vector equations in three vector unknowns (i.e., $\underline{a}_c, \underline{\omega}$, and $\underline{\dot{\omega}}$). True motion reproduction will require knowledge of these three vectors (i.e., 9 parameters) leading to the fact that **at least** three tri-axial transducers (i.e., $m \geq 3$) will be required during the field data collection phase. In the following development, it will be shown that, in the most general case, the use of three tri-axial transducers may be insufficient to reconstruct the motion. It will also be shown that using four appropriately positioned tri-axial transducers will eliminate this difficulty and, in fact, result in a much simpler mathematical representation.

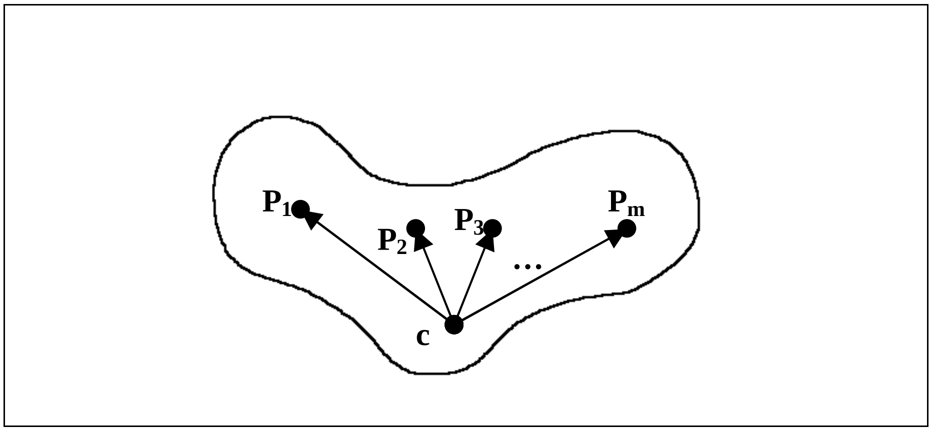


Figure 1 Generic Rigid Body with Transducers

Let's assume that there are three measurement points (i.e., $m = 3$). First, eliminate $\underline{a}_c$ from Equation (1) by developing the relative accelerations, arbitrarily using location 1as the point of reference.

$$\underline{a}_{j1} = \underline{a}_j - \underline{a}_1 = \underline{\dot{\omega}} \times \underline{r}_{j1} + \underline{\omega} \times \left(\underline{\omega} \times \underline{r}_{j1}\right), \quad j = 2,3,... \tag{2}$$

where, $\underline{r}_{j1} = \underline{r}_j - \underline{r}_1$. Next, cross $\underline{r}_{j1}$ into Equation (2) to obtain

$$\underline{r}_{j1} \times \underline{a}_{j1} = \underline{r}_{j1} \times \left(\underline{\dot{\omega}} \times \underline{r}_{j1}\right) + \underline{r}_{j1} \times \left(\underline{\omega} \times \left(\underline{\omega} \times \underline{r}_{j1}\right)\right), \quad j = 2,3,... \tag{3}$$

Observe that the LHS of Equation (2) represents quantities that are completely known. Now, the first term on the RHS of Equation (3) can be rewritten as

$$\underline{r}_{j1} \times \left(\underline{\dot{\omega}} \times \underline{r}_{j1}\right) = \left(\underline{r}_{j1} \cdot \underline{r}_{j1}\right)\underline{\dot{\omega}} - \underline{r}_{j1}\left(\underline{r}_{j1} \cdot \underline{\dot{\omega}}\right) = \underline{\underline{\vartheta}}_{j1} \cdot \underline{\dot{\omega}} \quad j = 2,3,...$$

where $\underline{\underline{\vartheta}}_{j1} = \left(\underline{r}_{j1} \cdot \underline{r}_{j1}\right)\underline{\underline{E}} - \underline{r}_{j1}\underline{r}_{j1}$ can be considered the contribution of a unit mass at location j to the inertia dyadic measured relative to location 1. Similarly, the second term on the RHS of Equation (3) can be rewritten as

$$\underline{r}_{j1} \times \left(\underline{\omega} \times \left(\underline{\omega} \times \underline{r}_{j1}\right)\right) = -\underline{\omega} \times \underline{r}_{j1}\left(\underline{r}_{j1} \cdot \underline{\omega}\right) = \underline{\omega} \times \left(\underline{r}_{j1} \cdot \underline{r}_{j1}\underline{\underline{E}} - \underline{r}_{j1}\underline{r}_{j1}\right) \cdot \underline{\omega} = \underline{\omega} \times \underline{\underline{\vartheta}}_{j1} \cdot \underline{\omega}, \quad j = 2,3$$

where in both expressions $\underline{\underline{E}}$ represents the unit dyadic. Thus, Equation (3) becomes

$$\underline{r}_{j1} \times \underline{a}_{j1} = \underline{\underline{\vartheta}}_{j1} \cdot \underline{\dot{\omega}} + \underline{\omega} \times \underline{\underline{\vartheta}}_{j1} \times \underline{\omega}, \quad j = 2,3,... \tag{4}$$

In general, the vectors in Equation (4) are all coordinatized in the same bases, therefore the vector operations can be replaced by their matrix equivalent, resulting in

$$\tilde{\underline{r}}_{j1}\underline{a}_{j1} = \underline{\underline{\vartheta}}_{j1}\underline{\dot{\omega}} + \tilde{\underline{\omega}}\underline{\underline{\vartheta}}_{j1}\underline{\omega}, \quad j = 2,3,... \tag{5}$$

where $\left(\tilde{\underline{\ }}\right)$ is a skew-symmetric matrix equivalent to a vector cross-product; e.g., if $\underline{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\tilde{\underline{b}} = \begin{bmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix}$$

Unfortunately, the individual contributions represented by $\underline{\underline{\vartheta}}_{j1}$ are rank deficient matrices and thus cannot be used independently to determine $\underline{\dot{\omega}}$. When the two equations (j=2,3) are combined, however, $\underline{\dot{\omega}}$ can be obtained from

$$\underline{\dot{\omega}} = \left[\underline{\underline{\vartheta}}_{21} + \underline{\underline{\vartheta}}_{31}\right]^{-1}\left[\left(\tilde{\underline{r}}_{21}\underline{a}_{21} + \tilde{\underline{r}}_{31}\underline{a}_{31}\right) - \tilde{\underline{\omega}}\left(\underline{\underline{\vartheta}}_{21} + \underline{\underline{\vartheta}}_{31}\right)\underline{\omega}\right] \tag{6}$$

since $\left[\underline{\underline{\vartheta}}_{21} + \underline{\underline{\vartheta}}_{31}\right]$ now represents the "specific" inertia dyadic of the resulting two particle system. It is well known [1] that the inertia dyadic of an *n*-particle system is full ranked provided the particles are not all along a line. By definition, particles distributed along a line will have zero inertia about the line and therefore the inertia dyadic of the system will be rank deficient. This is easily observed by considering the case of colinear relative position vectors (e.g., $\underline{r}_{31} = k\underline{r}_{21}$) which results in $\left\lfloor \underline{\underline{\vartheta}}_{21} + \underline{\underline{\vartheta}}_{31} \right\rfloor = (1+k)\underline{\underline{\vartheta}}_{21}$ which is rank deficient as previously stated. Vectors $\underline{r}_{21}$ and $\underline{r}_{31}$ are colinear *iff* $\underline{r}_1$, $\underline{r}_2$, and $\underline{r}_3$ define points along a line [2]. Intuitively, the closer the points defined by $\underline{r}_1, \underline{r}_2$, and $\underline{r}_3$ approach a straight line, the more ill-conditioned the coefficient matrix $\left\lfloor \underline{\underline{\vartheta}}_{21} + \underline{\underline{\vartheta}}_{31} \right\rfloor$ becomes. Similarly, if one point is significantly separated from the other two, then the matrix will also be ill-conditioned. This occurs because the two closely spaced points act as a single point, which along with the separated point forms a line. Issues relating to the conditioning of the coefficient matrix will be discussed in the Examples section.

At this point, it has been shown that if three transducers are utilized, then they must be distributed such that their relative position vectors are not colinear (i.e., the transducers cannot be placed along a line). In fact, it can easily be shown that this requirement applies to $m \geq 3$ transducers. It will now be shown that under certain conditions, three appropriately positioned tri-axial transducers may not be capable of uniquely determining the nine parameters ($\underline{a}_c$, $\underline{\omega}$, and $\underline{\dot{\omega}}$) required to reproduce the motion.

Again, eliminate $\underline{a}_c$ by considering the relative motion as described by Equation (2). Now consider the plane formed by $\underline{\omega}$ and $\underline{r}_{j1}$ (see Figure 2). The vector $\underline{\omega} \times \underline{r}_{j1}$ (which is the relative velocity $\underline{v}_{j1}$) is perpendicular to the plane defined by $\underline{\omega}$ and $\underline{r}_{j1}$; furthermore, the j[th] centripetal acceleration, $\underline{a}_{j1_{cent}} = \underline{\omega} \times (\underline{\omega} \times \underline{r}_{j1})$ is orthogonal to both $\underline{\omega}$ and $\underline{v}_{j1}$. Thus the three vectors, $\underline{\omega}$, $\underline{v}_{j1}$, and $\underline{a}_{j1_{cent}}$ form a dextral orthogonal set. If we now assume that $\underline{\dot{\omega}}$ is known (i.e., determined as described above), then the centripetal acceleration can be determined from equation (2) and used to determine $\underline{\omega}$. With this assumption, the "known" vectors in Figure 2 are $\underline{r}_{j1}$ and $\underline{a}_{j1_{cent}}$. Using these known vectors, the cross-product operator is used to develop $\underline{\omega}$ by first constructing $\underline{v}_{j1}$ and then cross into $\underline{a}_{j1_{cent}}$ to obtain $\underline{\omega}$; i.e.,

$$\underline{\hat{v}}_{ji} = \frac{a_{j1_{cent}} \times r_{j1}}{\left\| a_{j1_{cent}} \times r_{j1} \right\|} \tag{7a}$$

$$\left\lfloor \omega r_{j1} \sin(\theta) \right\rfloor \underline{\omega} = \underline{\hat{v}}_{j1} \times \underline{a}_{j1_{cent}} \tag{7b}$$

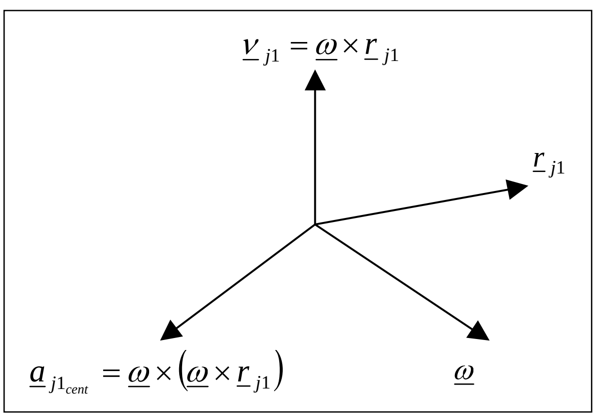


Figure 2. Vector Constructs for Angular Velocity

Using rather laborious successive substitution techniques (since $\underline{\dot{\omega}} = \dot{\omega}(\omega)$), Equation (7) could be used to uniquely determine $\underline{\omega}$ provided the relative position vectors $\underline{r}_{j1}$ are not all orthogonal to $\underline{\omega}$ (i.e., the angular velocity is perpendicular to the plane formed by the transducer locations). Should this situation exist, Equation (7a) yields the zero vector for $\underline{v}_{j1}$ since $\underline{r}_{j1}$ and $\underline{a}_{j1_{cent}}$ are colinear and thus a solution cannot be determined; however, by simply adding a non-coplanar fourth measurement location, this problem can be eliminated. Therefore, in the most general case, three transducers may be insufficient to determine the actual motion since three points will always form a plane whose normal may be colinear with the angular velocity. Thus, the minimum number of transducers required to uniquely determine the actual motion of a rigid body is four. In the next section the four-transducer case is discussed. The simplicity of the corresponding mathematics is also demonstrated.

MOTION RECONSTRUCTION USING 4 TRANSDUCERS

From Equation (1), it is observed that in order to reconstruct the accelerations obtained at the various transducer locations it is only necessary that the sum of the various contributions be known, not the individual contributions themselves (i.e., it is not necessary to determine tangential and centripetal accelerations independently). Therefore, assuming that the vectors are all coordinatized in the same basis system (e.g., a frame attached to the body), Equation (1) can be rewritten in matrix form as

$$\underline{a}_i = \underline{a}_c + \left[\tilde{\dot{\underline{\omega}}} + \tilde{\underline{\omega}}\tilde{\underline{\omega}}\right]\underline{r}_i = \underline{a}_c + \underline{\underline{\Omega}}\,\underline{r}_i$$

where the 3x3 coefficient matrix $\underline{\underline{\Omega}}$ contains the rotational parameters. In this form, there are now 12 unknowns to be determined (i.e., the three elements of $\underline{a}_c$ and the nine elements of $\underline{\underline{\Omega}}$), resulting in the need for at least four tri-axial transducers. As before, $\underline{a}_c$ is eliminated by considering relative accelerations. The resulting three relative acceleration equations can then be used to construct the following system of equations:

$$\left[\underline{a}_{21}\ \underline{a}_{31}\ \underline{a}_{41}\right] = \underline{\underline{\Omega}}\left[\underline{r}_{21}\ \underline{r}_{31}\ \underline{r}_{41}\right] \tag{8}$$

where $\underline{a}_{j1}$ and $\underline{r}_{j1}$ are as previously defined. The coefficient matrix of rotational parameters is then obtained as

$$\underline{\underline{\Omega}} = \left[\underline{a}_{21}\ \underline{a}_{31}\ \underline{a}_{41}\right]\left[\underline{r}_{21}\ \underline{r}_{31}\ \underline{r}_{41}\right]^{-1} \tag{9}$$

provided that the relative position vectors $\underline{r}_{j1}$ are linearly independent. Having determined $\underline{\underline{\Omega}}$, the acceleration of the reference point $\underline{a}_c$ is determined from any of the four transducer locations by

$$\underline{a}_c = \underline{a}_i - \underline{\underline{\Omega}}\,\underline{r}_i \tag{10}$$

Equations (9) and (10) can be used to uniquely determine the 12 parameters required to reconstruct the acceleration at any arbitrary point on the rigid body (e.g., the exciter drive points required to replicate the field scenario in a laboratory test). Note that even though over-determined linear measurements were required to implement the mapping algorithm, that the mapping process is still exact in the time domain within the assumptions of rigid body motion.

One may question whether this approach could be used with only three transducers. Since three measurement locations will produce at most nine unique pieces of information and there are actually12 unknown parameters, a unique solution is not possible. This result manifests itself in the fact that the relative position coefficient matrix for three transducer locations would be of a form $\left[\underline{r}_{21}\ \underline{r}_{31}\ \underline{r}_{23}\right]$, which has a rank of 2 since $\underline{r}_{23} = \underline{r}_{21} - \underline{r}_{31}$. Therefore, solutions could only be attempted using pseudo-inverse techniques and hence true motion reconstruction cannot be guaranteed.

Another issue that arises is that of transducer placement. Consider four transducers whose positions are defined by $\underline{r}_i$, i = 1,2,3,4. If the transducers are in a plane, their position vectors satisfy

$$\underline{r}_4 = l\underline{r}_1 + m\underline{r}_2 + n\underline{r}_3, \quad l + m + n = 1$$

The relative position vectors required in Equation (9) are

$$\underline{r}_{21} = \underline{r}_2 - \underline{r}_1,\ \ \underline{r}_{31} = \underline{r}_3 - \underline{r}_1, \text{ and } \ \underline{r}_{41} = \underline{r}_4 - \underline{r}_1 = l\underline{r}_1 + m\underline{r}_2 + n\underline{r}_3 - \underline{r}_1 = m\underline{r}_{21} + n\underline{r}_{31}$$

where $\underline{r}_{41}$ is clearly linearly dependent on $\underline{r}_{21}$ and $\underline{r}_{31}$. Thus, the coefficient matrix $\left[\underline{r}_{21}\ \underline{r}_{31}\ \underline{r}_{41}\right]$ would not be full rank and hence a unique solution would not be available. Obviously, there is no need to consider the case(s) in which any three (or four) transducers are colinear since by definition they must also be coplanar. In all other cases, the coefficient matrix will be full rank (there may however, be some conditioning issues). Thus, in addition to the non-colinear placement constraint the additional constraint in placing the transducers for the $m \geq 4$ case is that they are not all coplanar.

EXAMPLES

It was shown that for the $m = 3$ case, $\underline{\omega}$ could be uniquely determined provided that the relative position vectors $\underline{r}_{j1}$ are not all orthogonal to $\underline{\omega}$ (i.e., the angular velocity is perpendicular to the plane formed by the transducer points). However, there still exist numerical concerns associated with the inversion required in Equation (6) that are directly related to the positioning of the measurement transducers. A common metric used to study the numerical stability of a matrix inversion is the condition number. For this discussion, the condition number with respect to inversion is defined as the ratio of the largest to smallest singular values. A high condition number corresponds to a nearly singular matrix.

Consider the following example relating to conditioning for a generic $m = 3$ case. Three positions are arbitrarily chosen as $\underline{r}_1 = [2 \quad 0 \quad 0]^T$, $\underline{r}_2 = [1 \quad 0 \quad 0]^T$, and $\underline{r}_3 = [3 \quad 0 \quad \varepsilon]^T$ with ε representing the perpendicular distance from the line (x-axis). The relative position vectors and the resulting inertia dyadic are as follows:

$$\underline{r}_{21} = [-1,0,0]^T, \ \underline{r}_{31} = [1,0,\varepsilon]^T \text{ and } \left[\underline{\underline{\vartheta}}_{21} + \underline{\underline{\vartheta}}_{31}\right] = \begin{bmatrix} \varepsilon & 0 & -\varepsilon \\ 0 & 2+\varepsilon^2 & 0 \\ -\varepsilon & 0 & 2 \end{bmatrix}$$

Figure 3 shows the effects of moving the third location away from the x-axis. As ε is increased, the condition number is drastically reduced from infinity to approximately 8 before increasing again. This is expected since as ε increases, the plane formed by $\underline{r}_1$, $\underline{r}_2$ and $\underline{r}_3$ once again approaches a straight line (i.e., points 1 and 2 are closely spaced relative to point 3).

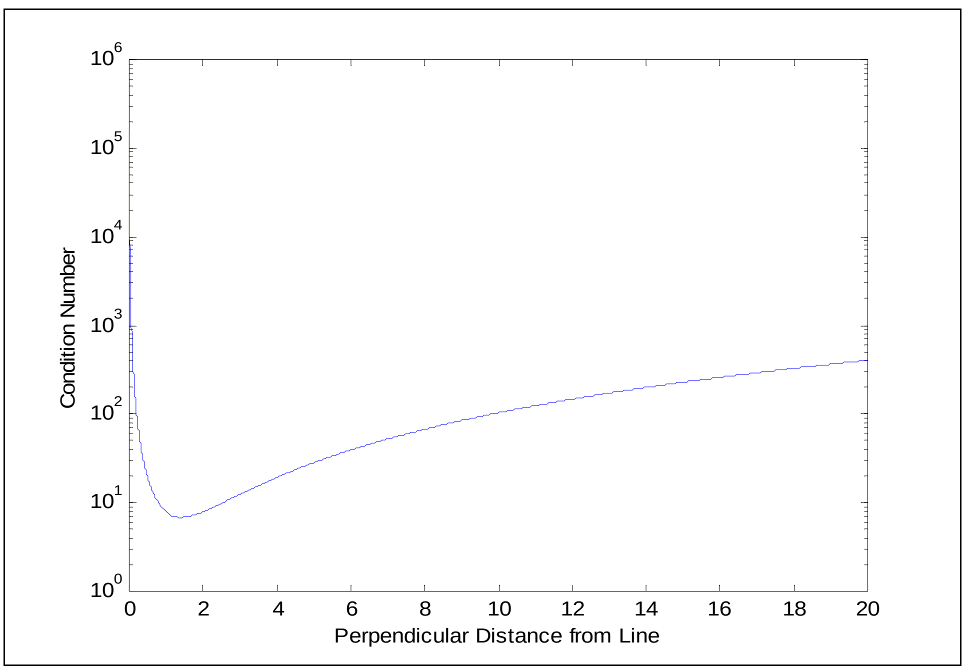
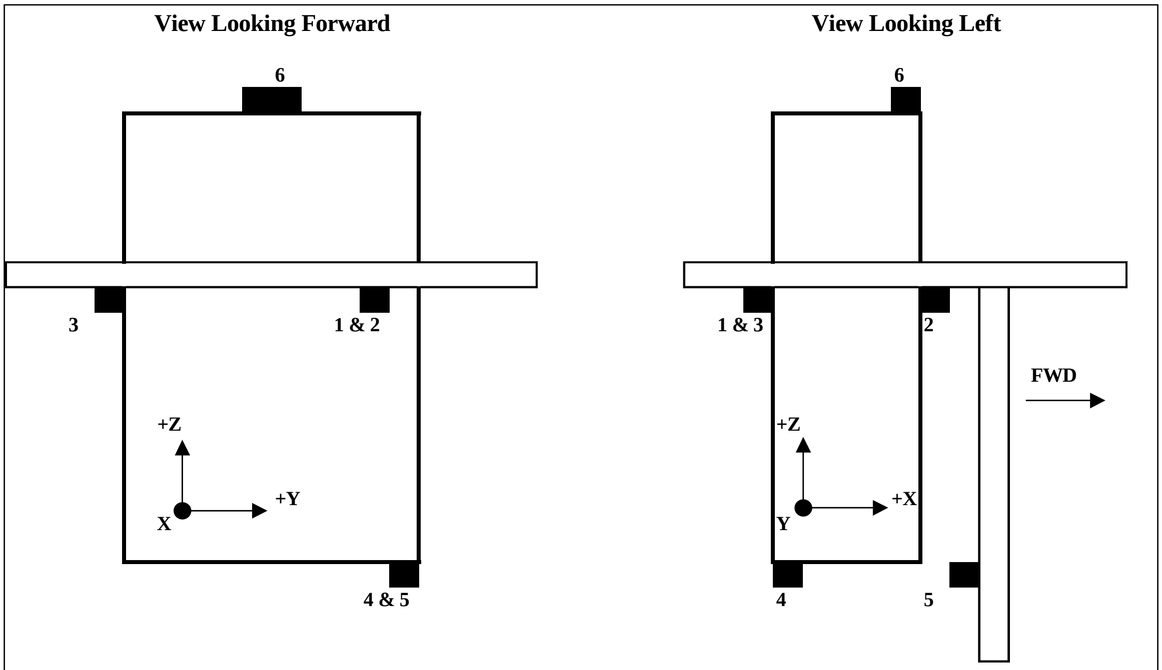


Figure 3. Relative Position Matrix Condition Number as a Function of Perpendicular Distance from a Line

To illustrate laboratory application of the concepts presented for the $m = 4$ case in the previous section, linear acceleration data acquired on a track vehicle will be considered (see Figure 4). The 6-DOF exciter used in this study was the TEAM Cube Model 3 [4]. Using the procedure presented in this paper, steps 1-4 can all be conducted in the field and therefore can be considered field pre-test procedures. Step 4 also introduces the concepts used to test the effectiveness of the mapping procedure in performing drive point control when using control systems that are not capable of processing over-determined measurements.



	(	X	(P) ,	Y	(P) ,	Z	(P))
1	(	-5	(-) ,	-2	(+) ,	14.5	(-))
2	(	10.5	(-) ,	3.5	(-) ,	14.5	(-))
3	(	-5.5	(-) ,	-14	(-) ,	14.5	(-))
4	(	0	(+) ,	0	(-) ,	0	(-))
5	(	8	(-) ,	-3.5	(+) ,	-1.75	(+))
6	(	8	,	-8.75	,	28	)

Position Vector Units = Inches
(P) = Accelerometer Polarity wrt Reference Coordinates Displayed
Rate Sensor = CCW is Positive

Figure 4: Transducer Placement

1. Five tri-axial accelerometers (locations 1-5) and a rate gyro (location 6) were placed as described in Figure 4. During the acquisition of the field data, transducers 1, 2, 3, and 5 were placed on a "rigid" bulkhead to minimize any local resonance effects; transducer 4 was attached to the test article, which was cantilevered from the bulkhead (i.e., transducer 4 violates the rigid body assumption!). Based on these considerations, the relative position matrix, as required in Equation (8), is formulated using the four rigid bulkhead locations.

$$\underline{\underline{R}} = \begin{bmatrix} \underline{r}_{21} & \underline{r}_{31} & \underline{r}_{51} \end{bmatrix} = \left[\begin{bmatrix} 10.5 \\ 3.5 \\ 14.5 \end{bmatrix} - \begin{bmatrix} -5 \\ -2 \\ 14.5 \end{bmatrix}, \begin{bmatrix} -5.5 \\ -14 \\ 14.5 \end{bmatrix} - \begin{bmatrix} -5 \\ -2 \\ 14.5 \end{bmatrix}, \begin{bmatrix} 8 \\ -3.5 \\ -1.75 \end{bmatrix} - \begin{bmatrix} -5 \\ -2 \\ 14.5 \end{bmatrix} \right] = \begin{bmatrix} 15.5 & -0.5 & 13 \\ 5.5 & -12 & -1.5 \\ 0 & 0 & -16.25 \end{bmatrix}$$

Note that $\underline{\underline{R}}$ is full rank and well conditioned (Condition Number =2.75), therefore its inverse is easily and accurately computed as:

$$\underline{\underline{R}}^{-1} = \begin{bmatrix} 0.0655 & -0.0027 & 0.0526 \\ 0.0300 & -0.0846 & 0.0318 \\ 0 & 0 & -0.0615 \end{bmatrix}$$

2. Design a fixture capable of integrating the test article onto the TEAM 6-DOF exciter. As is the case for performing any vibration test, care should be taken to avoid having critical fixture resonant frequencies in the test band of interest. In the example considered, fixture design is trivial since the maximum test frequency of interest is only 125 Hz. From the fixture design, it is now possible to provide exact positions of each of the six drive points. Assuming the test article is to be mounted to the 6-DOF exciter using the aforementioned fixture, the fixture and 6-DOF system can be treated as a single rigid body and all points can be mapped from a single global reference.

3. An algorithm (m-file) was written in MATLAB [3] to implement the methodology described by Equations (8-10). The m-file was structured to enable mapping to arbitrary locations (e.g., drive point locations). With such constructs, a fifth measurement point can be used in the field to investigate/validate the transducer locations used and the applicability the rigid body approximation. (Note that mapping back to any of the locations used by the algorithm to develop the map would simply produce a unity transfer function due to the rigid body assumption.) Although this process requires a fifth tri-axial measurement location, it is strongly recommended. For the example being considered, measurement locations 1,2,3, and 5 were used as reference locations for the mapping algorithm and location 4 was selected as the arbitrary verification location. The transfer function and comparison of Power Spectral Densities (PSD's) between the mapped data and measured data along the vertical axis at location 4 (the dominant degree of freedom for this data set) are shown in Figures 5 and 6, respectively. Note that the transfer function magnitude is near unity and phase is near zero degrees through approximately 125 Hz, which is the maximum frequency of interest for the test case. Similar results were obtained for the longitudinal and transverse axes of location 4, however, the bandwidth indicating rigid body characteristics (i.e., unity transfer function and zero phase shift between the mapped and measured data) held through only approximately 80 Hz. This result is expected at location 4 in the longitudinal and transverse axes due to the cantilevered mounting configuration of the test article, and not expected to be an issue at the rigid bulkhead locations.

4. In the laboratory, mount the test article to the 6-DOF exciter and place the four tri-axial accelerometers in the exact locations used in the original data acquisition exercise. Mount an in-axis accelerometer at each of the six drive points on the 6-DOF system. Use the time history associated with the degree-of-freedom of each drive point as the corresponding reference signal, and measure the data at each of the original measurement location for comparison to the original field data. Results and further discussion of these laboratory test results will be a follow-on effort to this paper.

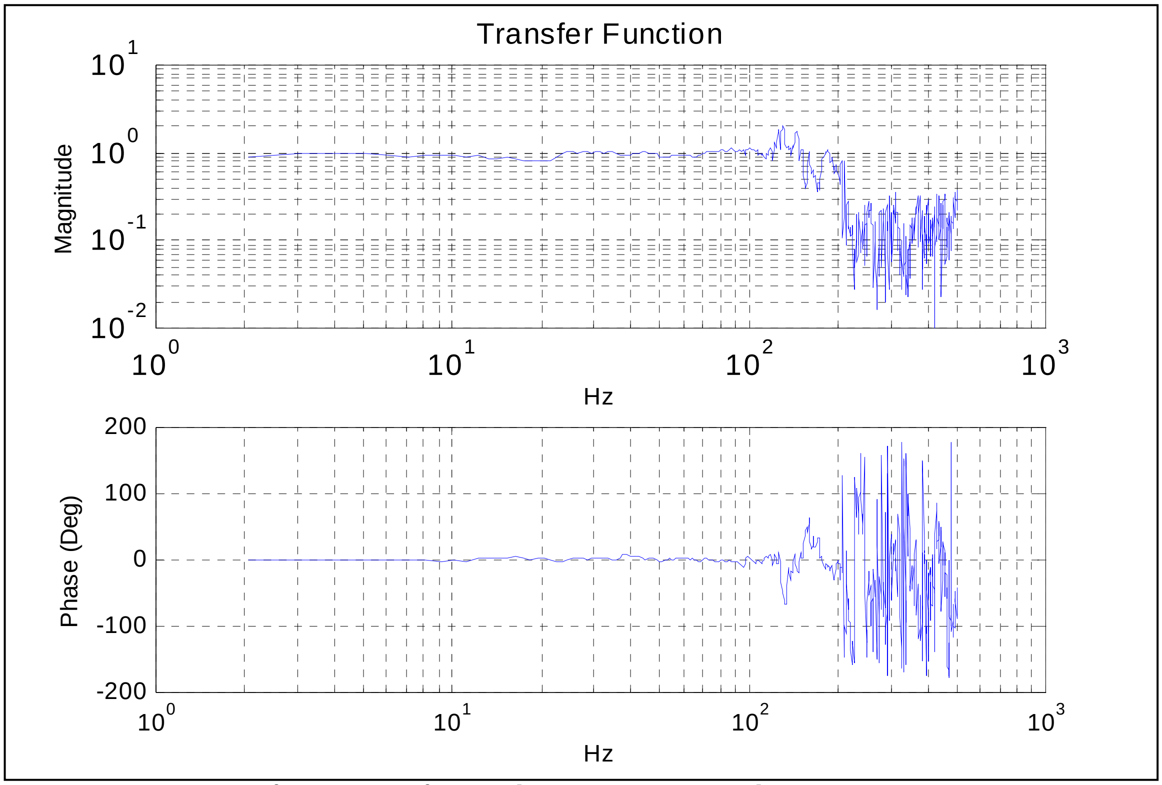


Figure 5: Transfer Function of Mapped Estimate & Measured Data at Location 4 Vert Axis

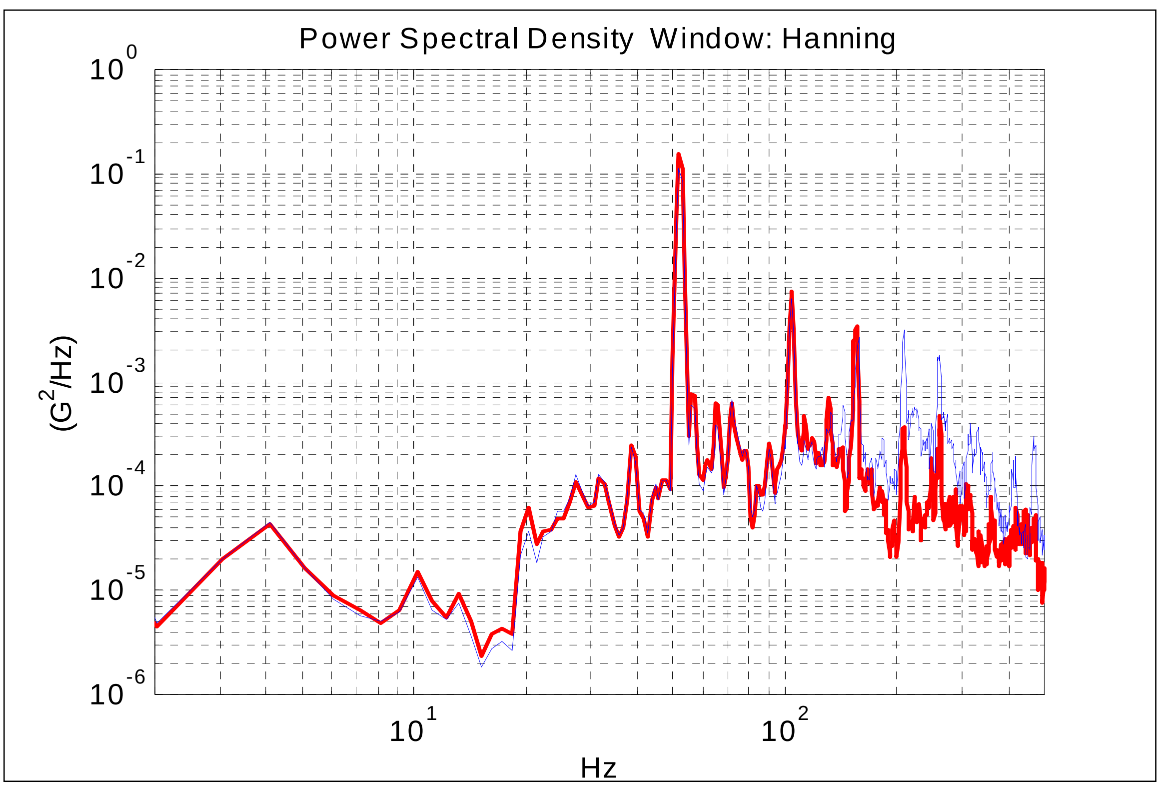


Figure 6: PSD Comparison of Measured Field Data (thick line) & Mapped Estimate (thin line) at Location 4 Vertical

CONCLUSIONS AND FUTURE WORK

In this paper, several considerations regarding the number of transducers and the placement of these transducers in acquiring data for 6-DOF vibration testing were investigated. Although the investigations were conducted in the time domain, the restrictions resulting from this analysis are also applicable to the frequency domain. The results presented here are physical kinematic constraints, which are independent of the domain in which the analysis is conducted. Thus, providing the transducer placement constraints are adhered to, one would generally expect good results using classical MIMO frequency domain techniques.

First, it was shown that under no circumstances can the *actual* 6-DOF motion be duplicated in the laboratory from data acquired with only two linear tri-axial transducers. Replication of the actual motion at the transducer locations would be possible, but there will be no guarantee that the motions at other arbitrary locations are being replicated. It was also shown that with proper consideration for the transducer positions (i.e., they cannot be colinear), three linear tri-axial transducers could provide unique mappings for 6-DOF motion. Unfortunately the mapping algorithm in the time domain is complex. It was also shown that great care must be exercised when making assumptions related to the angular velocities of the rigid body. For example, if the angular velocity is perpendicular to plane formed by the three transducers, a non-unique mapping condition is obtained. Since this non-unique condition resulted from the kinematics of the problem, one suspects that these conditions would also influence the results obtained from frequency domain analyses.

It was also shown that by using four linear tri-axial transducers and adhering to the basic position constraints discussed, the unique mappings for a rigid body can be easily obtained. Furthermore, the resulting mapping algorithm is greatly simplified over the case involving three transducers. Using actual field data, the utility of the methodology was verified up to the assumptions of rigid body (i.e., 125 Hz in the vertical direction and 80 Hz in the transverse and longitudinal directions). Based on these results, acceleration profiles at the drive points were developed for motion reconstruction in the laboratory.

The proposed simplified mapping methodology provides several key benefits starting with improved understanding of the fundamental of 6-DOF dynamics. It also provides an excellent tool to improve data acquisition planning, specifically the issue of transducer placement. In addition, feasibility analyses involving laboratory considerations (e.g., mapping to potential excitation points) may be performed quickly and accurately on site during the data acquisition phase. This provides the advantage of reconsidering transducer placement while still instrumented and in the field as opposed to the classical problem of discovering problems with transducer placement after the fact with no opportunity to recollect the reference data set.

Although the methodology developed in this paper is not applicable to non-rigid body dynamics, it can be used to provide rough estimates of the gain requirements in the portion of the spectrum influenced by structural flexibility. It is therefore recommended that this study be expanded to include the effects of structural flexibility in order that the full capabilities of the proposed methodology are explored. An investigation in the frequency domain may also prove worthwhile.

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