A 6-DOF Vibration Specification Development Methodology

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Abstract

Multiple degree of freedom (MDOF) excitation systems and MDOF vibration control systems continue to improve, and are now standard equipment in many dynamic test laboratories. This paper concentrates on the often overlooked process of determination of an input specification for such MDOF systems. A pair of generalized six-degree-of-freedom (6-DOF) vibration specification development (VSD) techniques are proposed, discussed, and illustrated through an example.

INTRODUCTION

The determination of an input specification for a MDOF vibration test is often confronted with issues, many of which may be related to insufficient measurements from which MDOF information is obtained. In this paper, it is assumed that field data are to be collected with the specific intent of development of a 6-DOF input specification. Thus, both auto-spectral density (ASD) and cross-spectral density (CSD) data will be known quantities yielding a fully populated spectral density matrix (SDM) for each measured event (each event is often referred to by Run Number in the following discussions).

An SDM is a three-dimensional matrix (row, column, and depth). In this discussion, the depth is frequency. The equations are written as two-dimensional matrices. It is understood that the calculations will be repeated at each level of depth (frequency lines for the discrete case). An SDM is also Hermitian $\Phi_{ij} = \Phi_{ij}^{*}$ at each level. As discussed by Smallwood[1,2], an SDM must be positive semi-definite to be physically realizable. In practice, such as in MDOF vibration control, the SDM must be positive definite. The VSD process entails manipulating the SDMs computed for each event under consideration and producing a representative SDM to serve as the input specification[3] for a MDOF vibration test. It is during this manipulation of SDMs that a non-positive definite reference is likely to be produced, if extreme care is not taken in the VSD process. Developing an MDOF VSD methodology that ensures a positive definite reference is a key feature in the process that follows.

KEYWORDS

Vibration specification development (VSD); multiple degree-of-freedom (MDOF); spectra density matrix (SDM), acceleration transformation, Cholesky decomposition, positive definite

DATA ACQUISTION/ANALYSIS CONSIDERATIONS

This section provides a short review of the key data acquisition and analysis considerations relative to the proposed MDOF VSD procedures. The associated references provide more detailed discussion on the topics of this section.

Acceleration Transformation: It has been shown[4,5] that the relationship between a set of linear acceleration measurements and corresponding rigid body motion in terms of the traditional Cartesian based DOFs (X,Y,Z,Rx,Ry,Rz) is given by:

$$\left[a_{k_j}\right] = \underset{(n\times 1)}{\begin{bmatrix} a_{1_j} \\ a_{2_j} \\ \vdots \\ a_{n_j} \end{bmatrix}} = \underset{(n\times 6)}{\begin{bmatrix} \underline{e}_j^T & -\underline{e}_j^T \left[{}^P\underline{r}_1^P \right]^{\times} \\ \underline{e}_j^T & -\underline{e}_j^T \left[{}^P\underline{r}_i^P \right]^{\times} \\ \vdots & \vdots \\ \underline{e}_j^T & -\underline{e}_j^T \left[{}^P\underline{r}_{n^*}^P \right]^{\times} \end{bmatrix}} \underset{(6\times 1)}{\begin{bmatrix} {}^P\underline{a}_o^P \\ {}^P\underline{\alpha}^P \end{bmatrix}} \quad i \in \left(1, 2, \ldots, n^*\right), \, j \in \left(x, y, z\right) \tag{1}$$

which is of the form:

$$\underset{(n\times1)}{\{\mathbf{a}\}_{\mathbf{meas}}} = \underset{(n\times6)}{\left[\overline{\mathbf{T}}_{\mathbf{a}}\right]}\underset{(6\times1)}{\{\mathbf{c}\}_{\mathbf{motion}}} \tag{2}$$

where parameters $\underline{e}_x^T = [1\ 0\ 0]$, $\underline{e}_y^T = [0\ 1\ 0]$, and $\underline{e}_z^T = [0\ 0\ 1]$ are defined as row selection matrices and $\left[{}^P\underline{r}^P\right]^\times$ is the skew operator equivalent of the cross-product; e.g. if ${}^P\underline{r}^P = \left[r_x\ r_y\ r_z\right]^T$, then $\left[{}^P\underline{r}^P\right]^\times = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}$. As defined by Fitz-Coy et. al.[4], the notation representing the matrix equivalent of a vector (i.e., a coordinatized vector quantity) is denoted as ${}^{(\ )}(\underline{\ \ })^{(\ )}_{(\ )}$ where the right superscript and subscript define the body and point of interest, respectively, and the left superscript denotes the coordinate frame in which the vector quantity was coordinatized; e.g., ${}^B\underline{r}_i^P$ denotes the i[th] point on body P coordinatized in the base frame F_B.

This construct provides a direct relationship of rigid body motion between a set of measurements, $\underset{(n\times1)}{\{\mathbf{a}\}_{\mathbf{meas}}}$ and the 6-DOF motion defined at a specific point of origin $\underset{(6\times1)}{\{\mathbf{c}\}_{\mathbf{motion}}}$. Observe that $\underset{(n\times6)}{\left[\overline{\mathbf{T}}_{\mathbf{a}}\right]}$ is entirely defined by knowledge of (i) placement, (ii) orientation, and (iii) utilized signals of the accelerometers. It was also shown[4,5] that if $\underset{(n\times6)}{\left[\overline{\mathbf{T}}_{\mathbf{a}}\right]}$ is of full rank, equation 2 can be manipulated to compute the motion DOFs as follows:

$$\begin{aligned}
&\{\mathbf{a}\}_{\mathrm{Meas}} = \overline{\mathbf{T}}_{\mathrm{a}}\{\mathbf{c}\}_{\mathrm{Motion}} \\
&\overline{\mathbf{T}}_{\mathrm{a}}^{\mathrm{T}}\{\mathbf{a}\}_{\mathrm{Meas}} = \overline{\mathbf{T}}_{\mathrm{a}}^{\mathrm{T}}\overline{\mathbf{T}}_{\mathrm{a}}\{\mathbf{c}\}_{\mathrm{Mot}} \\
&\left[\overline{\mathbf{T}}_{\mathrm{a}}^{\mathrm{T}}\overline{\mathbf{T}}_{\mathrm{a}}\right]^{-1}\overline{\mathbf{T}}_{\mathrm{a}}^{\mathrm{T}}\{\mathbf{a}\}_{\mathrm{Meas}} = \left[\overline{\mathbf{T}}_{\mathrm{a}}^{\mathrm{T}}\overline{\mathbf{T}}_{\mathrm{a}}\right]^{-1}\overline{\mathbf{T}}_{\mathrm{a}}^{\mathrm{T}}\overline{\mathbf{T}}_{\mathrm{a}}\{\mathbf{c}\}_{\mathrm{Mot}} \\
&\left[\overline{\mathbf{T}}_{\mathrm{a}}^{\mathrm{T}}\overline{\mathbf{T}}_{\mathrm{a}}\right]^{-1}\overline{\mathbf{T}}_{\mathrm{a}}^{\mathrm{T}}\{\mathbf{a}\}_{\mathrm{Meas}} = \{\mathbf{c}\}_{\mathrm{Motion}} \quad \textit{Defining}:\ \mathbf{T}_{\mathrm{a}} = \left[\overline{\mathbf{T}}_{\mathrm{a}}^{\mathrm{T}}\overline{\mathbf{T}}_{\mathrm{a}}\right]^{-1}\overline{\mathbf{T}}_{\mathrm{a}}^{\mathrm{T}} \quad \textit{yields}: \\
&\{\mathbf{c}\}_{\mathrm{Motion}} = \left[\mathbf{T}_{\mathrm{a}}\right]\{\mathbf{a}\}_{\mathrm{Meas}}
\end{aligned} \tag{3}$$

Matrix $[\mathbf{T}_{\mathbf{a}}]$ is commonly referred to as the Moore-Penrose pseudo inverse of $\left[\overline{\mathbf{T}}_{\mathbf{a}}\right]$. As discussed in the introduction section of this paper, it is assumed that field data is to be collected with the specific intent of developing a 6-DOF vibration test input specification. Therefore, transducer placements will need to be geometrically spaced such that $\underset{(n\times6)}{\left[\overline{\mathbf{T}}_{\mathbf{a}}\right]}$ is of full rank. In addition, it is often of benefit to over-determine the number of measurement locations to reduce the probability of having an ill-conditioned matrix resulting from inadvertently placing one of the measurement accelerometers directly on a modal node. Use of 3 or 4 non-collinear trixial accelerometers has proven to be sufficient in most cases to avoid such problems.

In the process that follows, the final reference SDM is defined in terms of the six motion DOFs relative to a user-defined "origin" at the measurement point of interest. Although it is possible to develop the specification SDM in terms of linear DOFs only, the process is complicated in that the dimension of the reference SDM will correlate directly to the number of measurement points (e.g., if four trixial accelerometers are employed in the field measurement, the resulting SDM, in terms of the linear DOFs measured, would be [12×12×depth] as opposed to [6×6×depth] for the motion DOF case). Working with and understanding the dynamics of a 6×6 matrix is difficult enough, so avoiding higher dimension situations is advantageous. Another disadvantage of using only linear motion as the reference SDM is that the rotational DOF information will not be observable as it will be defined in terms of phase and amplitude differences between the linear measurements. In addition, if the reference SDM were to be defined in terms of the linear measurements only, placement of the control points in the laboratory would be forced to be exactly correlated to the field data positions. By establishing a reference SDM in terms of the motion DOFs relative to a user-selected origin, the laboratory technician has flexibility in control point selection as long as the points of interest are identified with respect to the same origin as selected in the field measurements.

Spectral Density Matrix: The data structure manipulated and the final product of the MDOF VSD discussed in this paper will be in the form of a spectral density matrix. A spectral density matrix is a three-dimensional matrix. At each frequency line (the 3rd index) the matrix is a square complex matrix. Each diagonal element is the autospectrum of the corresponding element. An element in the SDM is loosely defined as:

$$G_{ji}(k) = 2\lim_{T\to\infty}\frac{1}{T}\mathrm{E}[X_j(k,T)X_i^*(k,T)]$$

where

$G_{ji}(k)$ is the cross spectral density between the j'th and i'th random processes

$X_j(k,T)$ and $X_i\ (k,T)$ are the discrete Fourier transforms of the time histories

k is the frequency index

If $i = j$ the spectrum is called the autospectrum or the power spectrum. In reality, we seldom know the true spectral density; we just have an estimate. Some authors define the elements as:

$$G_{ij}(k) = 2\lim_{T\to\infty}\frac{1}{T}\mathrm{E}[X_i^*(k,T)X_j(k,T)]$$

The SDM matrix is Hermitian positive definite.

Hermitian Matrix: A matrix, $\mathbf{A}$, is Hermitian if the diagonal elements are real positive numbers and the corresponding off-diagonal elements are complex conjugate pairs:

$$a_{ii} = \text{positive real number}$$

$$a_{ji} = a_{ij}^* = conj(a_{ij})$$

where a_{ji} is the element form j'th row, i'th column of $\mathbf{A}$.

Note: All valid spectral density matrices are Hermitian.

Cholesky Decomposition: The MDOF VSD process discussed in this paper takes advantage of a key property of the Cholesky decomposition[6]. Specifically, Matrix $\mathbf{A}$ is Hermitian (or real symmetric) and Positive Definite if and only if $\mathbf{A}$ can be written in the Cholesky decomposition:

$$\mathbf{A} = \mathbf{CC}^H \ or\ (\mathbf{A} = \mathbf{CC}^T),$$

where $\mathbf{C}$ is lower triangular and $\langle C \rangle_{ii} \neq 0$ for all i .

Miner-Palmgren Hypothesis: In the simplest terms, the Miner-Palmgren Hypothesis (Miner's rule) is a set of mathematical equations used to scale vibration spectra levels and their associated test times. Miner's rule provides a convenient means to analyze fatigue damage resulting from cyclical stressing. The mathematical expression and variable descriptions for this technique are illustrated in equation 4

$$\frac{t_2}{t_1} = \left[\frac{S_1}{S_2}\right]^M \quad (4)$$

where:

t_1 = equivalent test time

t_2 = in-service time for specified condition

S_1 = severity (rms) at test condition

S_2 = severity (rms) at in-service condition

[The ratio S_1/S_2 is commonly known as the exaggeration factor.]

M = a value based on (but not equal to) the slope of the *S-N* curve for the appropriate material where S represents the stress amplitude and N represents the mean number of constant amplitude load applications expected to cause failure. For the MDOF VSD work described here, the default of M = 7 was selected per reference 7.

MDOF VSD PROCESS OUTLINE

Having reviewed the data acquisition and analysis requirements, this section is dedicated to defining the steps in the two MDOF VSD methodologies proposed in reference 8 and formally presented in this paper. Many of the details will be similar to 1-DOF VSD as discussed in reference 7. Additional details on the SDM and Cholesky domain configurations employed in both methods are found in references 1 and 9.

Outline - Method I (SDM Domain)

Step 1: Prepare to convert field measurements into motion DOFs.

- Identify position vectors $r_1 - r_n$ and row selection vectors e_j as defined for equation 1 corresponding to the field measurements.
- Identify the mission scenario.
- Identify the frequency bandwidth of interest.
- Identify the sampling frequency of the field measurements.

Step 2: Transform the field measurements into motion DOFs per equation 3 for each Run identified in the mission scenario.

Step 3: Compute the SDM for each run identified in Step 2. The dimension of the resulting SDMs, which will be [6×6×d] where d is the number of spectral lines being considered, addresses the frequency bandwidth of interest.

- Since the SDM is computed from measured field data, it should be positive definite; however, a check may be run to verify that each individual SDM is positive definite. This serves as an excellent data quality check.
- Refer to the guidance in Step 7 if minor corrections are required to force an individual SDM to be positive definite.

Step 4: Convert the CSD terms (the off-diagonal terms of the SDM) into a normalized form in which the magnitude squared of the cross terms correlates to the ordinary coherence while leaving the phase unchanged.

- This is accomplished by normalizing (dividing) the CSD terms by $\sqrt{G_{xx}G_{yy}}$.
- While it is not absolutely necessary to conduct this step, it is often easier to understand the physical meaning of the CSD terms when viewed in terms of phase and coherence.

Step 5: Either organize all of the SDMs for the Runs of interest into a logical structure or merge the SDMs into one file of known matrix structure such as [SDM_Run1,SDM_Run2….SDM_Run*N*] to optimize the conduct of the basic statistics.

Step 6: Compute a weighted average SDM of the *N* SDMs of Step 5.

- It is critical that the weighted average be performed on the true complex CSD terms (**not** the normalized SDM).
- The weighting factor on the average will be directly correlated to the mission scenario identified in Step 1. If the individual Runs are positive definite, the resulting average should also be positive definite. However, numerical issues may yield non-positive definite results. To minimize numerical issues, average only the lower triangular portion of the SDM and fill in the upper triangular portion of the SDM by taking advantage of the Hermitian structure of the matrix.[10]
- Any type of enveloping operation should be avoided as it is highly likely to yield a non-positive definite result.

Step 7: As SDM data is manipulated through activities such as averaging, it is advisable to verify that the results remain positive definite as expected. As discussed earlier, occasional numerical issues may be of concern in some instances. If required, force the SDM computed in Step 6 to be positive definite. This is done by systematically reducing the magnitude of the cross spectral density terms until the Cholesky decomposition is possible at each depth (spectral line) of the SDM. (If required, this process may be somewhat conservative in its reduction of the coherence between DOFs in that the systematic reduction of cross-term magnitudes is applied to each cross term equally).

Step 8: Scale the diagonal terms of the autospectra (the diagonal terms of the SDM) resulting from Step 7 to the maximum rms level of each of the N SDMs in Step 5 on an individual DOF basis using Miner-Palmgren.

- Observe that a new total test time will be computed for each DOF and that it highly probably that the resulting test times for each DOF will not be the same.
- Since the magnitudes of the autospectra are being increased while not modifying the cross-spectral density terms, the resulting scaled SDM should still be positive definite. However, as discussed in Step 7, it is always a good idea anytime an SDM is manipulated to verify that the resulting SDM remains positive definite.

Step 9: Review the test time associated with each DOF resulting from Step 8 and select a reasonable test time to which the entire SDM may be referenced.

- Just as in the case of a 1-DOF VSD development, one should consider the general guidance to keep the final test amplitudes resulting from time compression no more than 3 dB above the maximum measured field data. Once a test time is selected, reapply Miner-Palmgren as required per DOF. Again, make sure the resulting SDM is positive definite and modify as required per Step 7.

Step 10: Scale the results from Step 9 up by 3 dB to account for uncontrolled variables such as fleet variations and scenario conditions not considered in the mission scenario. The resulting SDM and the test time association per Step 9 define the final specification.

- This is accomplished by pre- and post-multiplying the SDM by the square root of the ratio of the desired scaling factor as:
  $S_{Ynew} = S_s S_{Yold} S_s$ (e.g., to scale the SDM ASD terms by 3 dB while keeping the phase and ordinary coherence the same, the diagonal terms of S_s would be defined as $S_{s,ii} = \sqrt{2}$).

Outline – Method II (Cholesky Domain)

Steps 1-4: Correlate directly to Method I Outline.

Step 5: Perform a Cholesky decomposition on the individual SDM associated with each Run in the mission scenario.

- Since each individual Run was based on a physical event, the individual SDMs should be positive definite, thereby making the Cholesky decomposition possible. (All Runs would have been tested to verify each was positive definite or corrected as required per Step 3).

- Either organize all of the lower triangular matrices resulting from the Cholesky decomposition for the Runs of interest into a logical structure or merge the matrices into one file of known matrix structure such as [CHOL_Run1,CHOL_Run2….CHOL_Run*N*] to optimize the conduct basic statistics.

Step 6: Compute a weighted average lower triangular matrix of the *N* Cholesky decompositions of Step 5.

- The weighting factor on the average will be directly correlated to the mission scenario identified in Step 1. Note that the resulting average will still consist of positive Eigen values, implying that when converted back into the SDM format, the result will be positive definite.
- Once converted back into the SDM domain, the resulting CSD terms will generally be highly comparable to the average CSD values computed in Step 6 of Method I. However, the rms levels of the ASD terms will not be the same. In addition, the spectral shape of the ASD terms will generally have been slightly modified.

Step 7: Rescale the ASD terms of the SDM resulting from Step 6 to match the rms levels of those in Method 1 Step 6.

- Convert the CSD terms (the off-diagonal terms of the SDM) into a normalized form in which the magnitude squared of the cross terms correlates to the ordinary coherence while leaving the phase unchanged. (Again, while it is not absolutely necessary to conduct this step, it is often easier to understand the physical meaning of the CSD terms when viewed in terms of phase and coherence).
- The resulting SDM phase and coherence are expected to be very similar to those of the averaged field data produced in Method I. The ASD terms spectral shapes are expected to be slightly different (i.e. < 3dB per spectral line).

Steps 8-10: Correlate directly to Method I Outline.

EXAMPLE

To illustrate the process discussed above, a simple example was derived (Method I is addressed first). Using an available wheeled vehicle, the input to an onboard missile storage rack was instrumented as shown in Figure 1. The transducer at the center of Figure 1 was placed at the user-defined origin, position [0,0,0], in terms of a Cartesian coordinate system. In a traditional right-hand orientation, the forward direction of the vehicle was defined as the positive x-axis, towards the vehicle driver's side was considered positive y-axis, and through the vehicle roof was considered the positive z-axis. All transducers are referenced in terms of their relative placement to the origin as discussed previously in the acceleration transformation section of this paper.

Figure 1. Transducer placement (input to missile rack).

Method I - Having established a clear coordinate system definition, the key parameters discussed in Step 1 are identified. In distance units of inches, the positions of the four corner accelerometer locations used in this example are defined as $r1 = [-17,-6,0]'$, $r2 = [-17,6,0]'$, $r3 = [17,-6,0]'$, $r4 = [17,6,0]'$. For convenience, the instrumentation team placed the tri-axial transducers such that the channel used to measure the y-axis motion was actually 180 degrees out of phase with respect to the referenced coordinate system. This issue is addressed by simply defining row selection vectors as $e_x^T = [1,0,0]$, $e_y^T = [0,-1,0]$, $e_z^T = [0,0,1]$. The field data were sampled at 4096 Hz and the bandwidth of interest is 500 Hz. For the example at hand, a mission scenario was established using a Beta distribution as discussed in reference 7, and is illustrated in Table 1. Allowing for the time associated with speeds below 5 mph, the total mileage represented is approximately 300.

Table 1. Mission Scenario.

Road Classification	**Speed (MPH)**	**Time (Hrs)**	**Distance (Miles)**
Embedded Rock	5	.690	3.45
Embedded Rock	10	1.545	15.45
Embedded Rock	15	.737	11.05
Cross Country	10	5.18	51.80
Cross Country	20	6.332	126.64
Cross Country	30	2.002	60.06
Radial Washboard	5	.811	4.055
Radial Washboard	7	1.841	12.88
Radial Washboard	10	1.183	11.83

The field data were then converted into motion DOFs using equation 3, and subsequently transformed into the frequency domain in the form of an SDM per Run as described in Steps 2 and 3 respectively. Each SDM was tested per the Cholesky decomposition property and verified to be positive definite. Each SDM was then normalized as suggested in Step 4 to allow the analyst to review the degree of coherence between DOFs.

Per Step 5, the SDMs were configured into a convenient structure to allow statistical analysis. MATLAB was employed in this example and the data were configured as SDM_all= [SDM_Run1,SDM_Run2….SDM_Run8]. Observe that only eight of the nine Runs identified in the scenario are being considered. In reviewing the field data, the 5-mph radial washboard data were significantly lower than the other Runs, determined to have no effect on fatigue, and were not considered in computing the basic statistics of the ensemble. Next, per Step 6, a weighted average was computed in terms of the time per road condition as defined in Table 1. This average should be computed in complex CSD terms, not the normalized SDM. The resulting weighted average SDM was then tested at each spectral line to establish whether or not the positive definite criterion was met. Figure 2 illustrates the weighted average SDM. Taking advantage of the Hermitian property of an SDM, Figure 2 is laid out such that the lower triangular section represents the phase between DOFs, the upper triangular portion represents the square root of the ordinary coherence, and the diagonal terms are the ASDs of the six rigid-body DOFs. Although too small to review in detail on a single page as shown, the coherence plots are all scaled between .1 and 1.0. This is to illustrate there is some level of coherence, particularly below 100 Hz in the example at hand, between DOFs. Using the VSD process proposed, the analyst will try to keep as much coherence in the final specification as possible, while still ensuring the final result is positive definite.

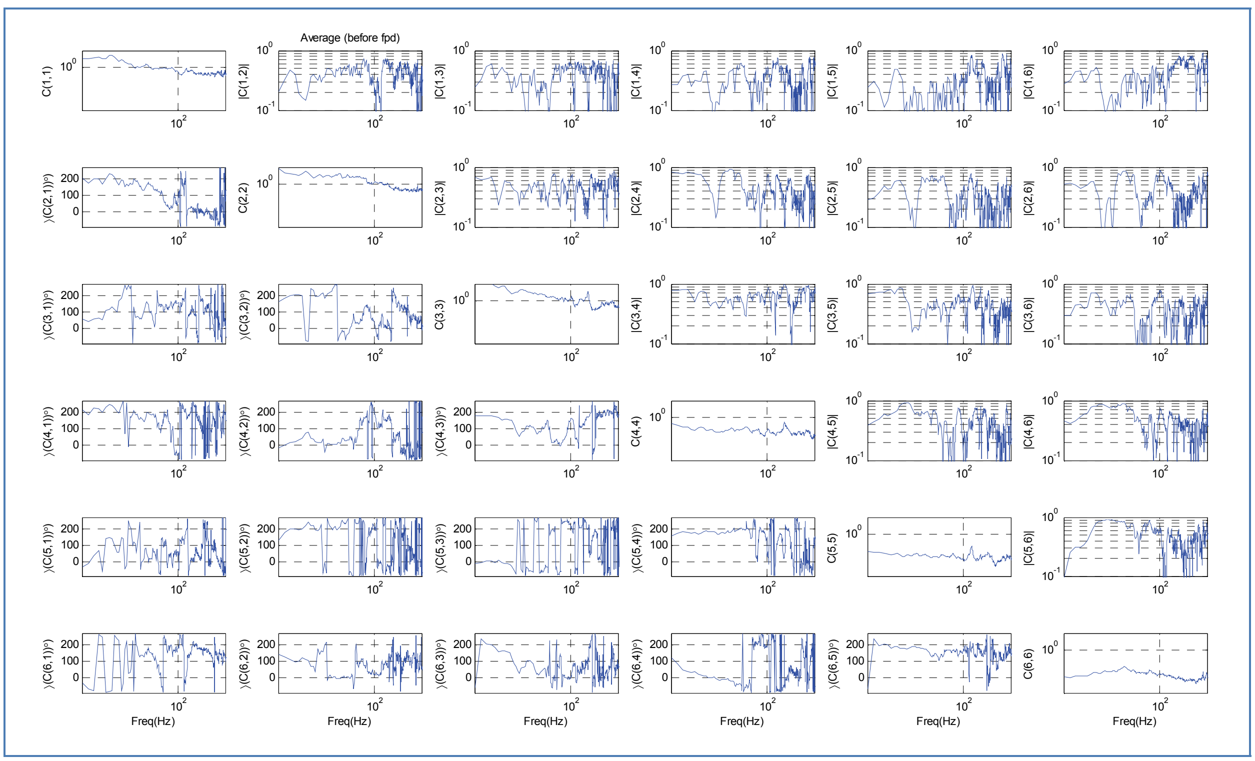

Figure 2. Normalized Weighted Average SDM.

To address the possibility of having to deal with non-positive definite results, a utility was written that gradually and equally reduces the magnitudes of the cross spectral density terms until the positive definite criterion is met per Step 7. This technique actually reduces the cross term magnitudes of some

CSDs more than is required. Addressing this potential shortcoming is one of the motivations for developing Method II.

At this point, per Step 8, the rms level was computed for each ASD (diagonal SDM Entry) over the bandwidth of interest (3-500 Hz in this example). Each ASD was then scaled to the level of the maximum rms level via equation 4. Per Step 9, the new test times associated with each ASD were also documented. As expected, the new times associated with each DOF were no longer the same. Since the VSD effort is designed to yield a simultaneous 6-DOF reference, it will be necessary to choose a common test time and rescale all ASD entries to the selected test duration. For the example at hand, a test duration of 15 min was selected. As always when selecting compressed test durations, one should not exaggerate the ASD power levels by more than 2:1. Of course, when dealing with 6 ASD terms this is not always possible. In such cases, the analyst should avoid increasing the dominant DOFs or DOFs with known structural shortcomings by more than 3 dB above maximum measured ASD levels.

The terms comprising the SDM were based on average ASD and CSD estimates, in contrast to the guidance provided in reference 7, in which the ASD levels carried through the calculations of a 1-DOF VSD were actually based on an ASD computed as the sum of a Mean ASD and standard deviation computed on a per-spectral-line basis. Working directly with the mean ASD levels is intended to avoid excessive conservatism in the VSD process. Conservatism intended to address uncontrolled variables such as fleet variations and conditions not considered in the mission scenario are addressed by a single scalar (+3 dB in this example) in Step 10. The analyst can modify the final conservatism level based on knowledge of the specific VSD effort.

The final reference SDM produced by Method 1 is shown in Figure 3. Observe that the phase and coherence terms are essentially unchanged from that of the average SDM of Figure 2.

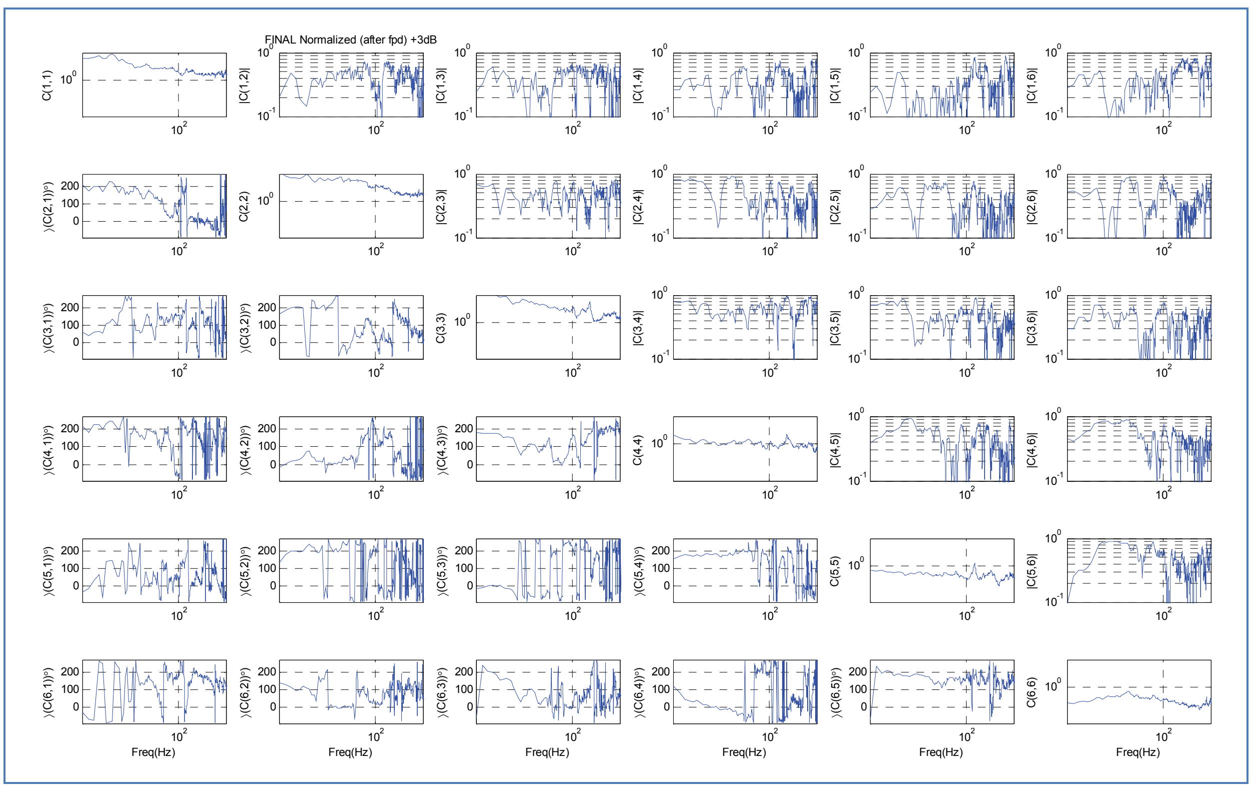

Figure 3. Method I Normalized Reference SDM.

Method II – The first four steps of Method II correlate directly to that of Method I. The major deviation in Method II is that all averaging will be computed in the Cholesky domain. In Step 5, Cholesky decompositions are carried out on the individual SDMs associated with each Run in the mission scenario. Since each individual Run was based on a measured physical event, the individual SDMs were positive definite as expected, thereby making the Cholesky decomposition possible. (If a given Run fails the Cholesky decomposition and all measurement locations and relative polarities are verified, investigate the spectral lines at which the decomposition fails. If the decomposition is failing at only a few spectral lines, it may be possible to salvage the measurement employing the CSD magnitude reduction techniques proposed in Method I). The Cholesky domain data were then organized into a convenient structure for statistical analysis. As in Method I, Matlab was used to compute the weighted averages and the Cholesky domain data were organized as: CHOL_all=[CHOL_Run1, CHOL_Run2…..,CHOL_Run*N*]. In Step 6, a weighted average in terms of the time per road condition as defined in Table 1 was computed over the lower triangular matrix of the eight Cholesky decompositions of Step 5. The weighted average was then converted back into the SDM domain. As expected, the coherence characteristics of the resulting SDM were comparable with those of Figure 2, and the rms levels of the ASD terms required rescaling per Step 7. Steps 8-10 were carried out directly as stated in the Method I outline.

The reference SDM resulting from Method II (Figure 4) yielded similar phase and coherence characteristics to that of the reference SDM resulting from Method I (Figure 3). Note that both Methods took advantage of averaging only the lower triangular terms, avoiding potential numerical issues, thereby not requiring the SDM to be forced positive definite in a manner that would result in lowering the coherence in a more conservative manner than required.

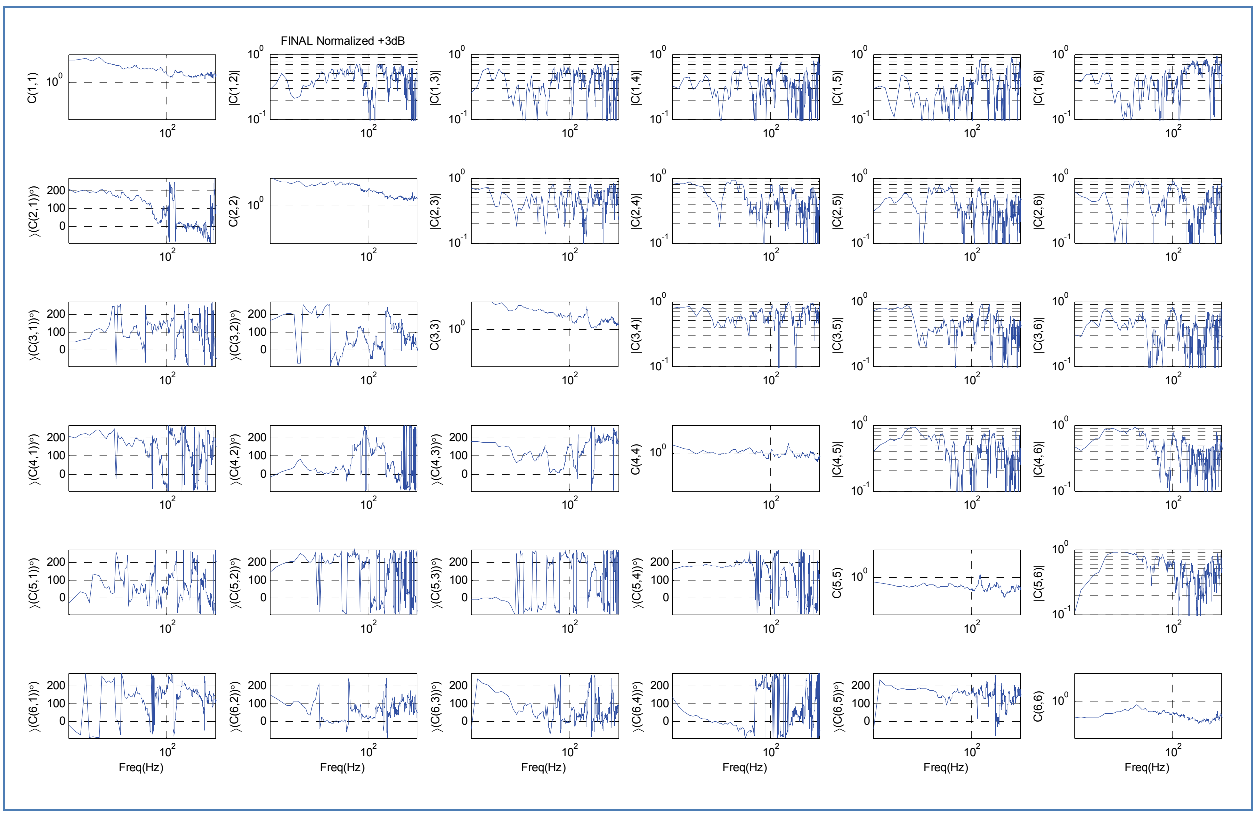

Figure 4. Method II Reference SDM.

ASD Comparisons – Next, the minor spectral shape deviations between the ASD resulting from the two VSD methods discussed will be illustrated. Figures 5 and 6 show the ASD references for the z-axis (vertical) and rotation about z-axis (Rz) respectively, as produced from both VSD methods. The ASD references are superimposed with the raw (unexaggerated) reference data from which the specifications were created. Observe that the ASD shapes envelope the field data without excessive conservatism.

As stated previously, the test duration for the Reference SDM yielded by both Methods in this example was established to be 15 min. Clearly, as illustrated in Figures 5 and 6, the associated ASD references are highly correlated.

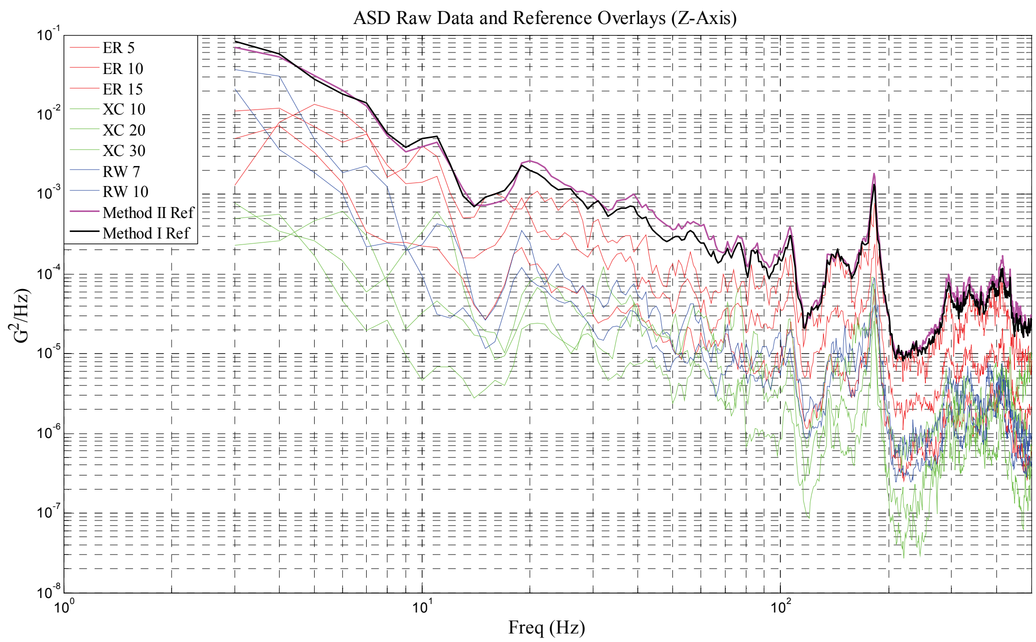

Figure 5. ASD references for the z-axis.

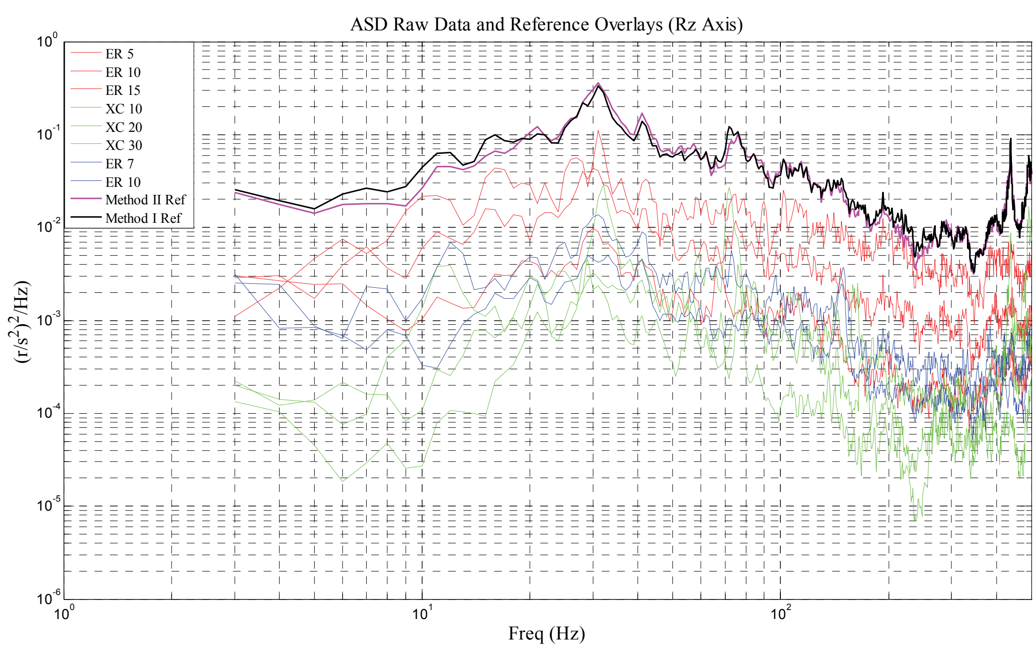

Figure 6. ASD references for rotation about z-axis (Rz).

DISCUSSION AND CONCLUSION

Two techniques were introduced for consideration in establishing an input specification for a MDOF system. It was shown that simple enveloping techniques are not appropriate when considering CSD terms, due to the sensitivity of such operations associated with maintaining a physically realizable reference. The resulting SDM references yielded through the process outlined are fully populated SDMs. Importing the fully populated SDM into the MDOF control system in an efficient manner is essential due to the volume of information involved.

While synthesizing a drive signal with CSD characteristics of the field data is desired, it is recognized that the mechanical impedance of the laboratory configuration is highly unlikely to match that of the field data. Therefore, it will be difficult to maintain CSD characteristics across the spectral bandwidth of interest, and thus the control hierarchy will generally place emphasis on the ASD terms. Also, it is not uncommon in MDOF tests for a specific mechanical degree of freedom to consist of a very small percentage of the composite energy across all mechanical degrees of freedom. In such cases, the associated error for the low DOF will often be higher than the desired test tolerances, and it may be necessary to consider global test tolerances.

Care was taken in the examples provided to limit conservatism in the VSD process. Conservatism is cumulative across degrees of freedom and, if not managed carefully, will yield test levels significantly higher than the measured environment. Unlike the common technique of essentially adding 3dB to all measurements prior to conducting averaging or enveloping techniques in the 1-DOF arena per reference 7, all weighted averages in the 6-DOF examples shown were based on raw averaged data. Conservatism to account for variables such as fleet variability and mission scenario omissions were added in the final step. Magnitude amplification associated with time compression techniques was limited to no more than maximum measured levels. Also, on the subject of tolerances, one may find it reasonable to define phase and coherence tolerances over only a portion of the test bandwidth. In the example provided, the coherence dropped off considerably at frequencies above 50 Hz. Since the phase term is essentially a random variable for low coherence, setting tolerances for frequencies greater than 50 Hz would not be recommended for the example shown.

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